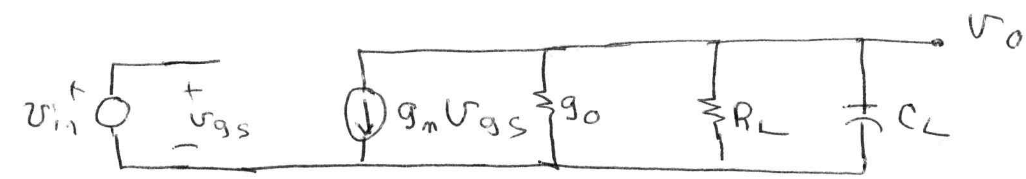
EE 330 Homework 13

Solutions

Problem 1



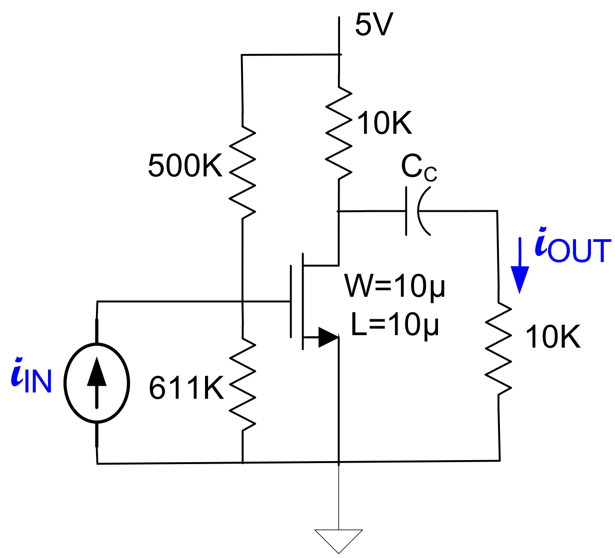
Summing currents on v_o node (with $G_L = 1/R_L$)

$$v_o \left(sC_L + G_L + g_o \right) + g_m v_i = 0$$

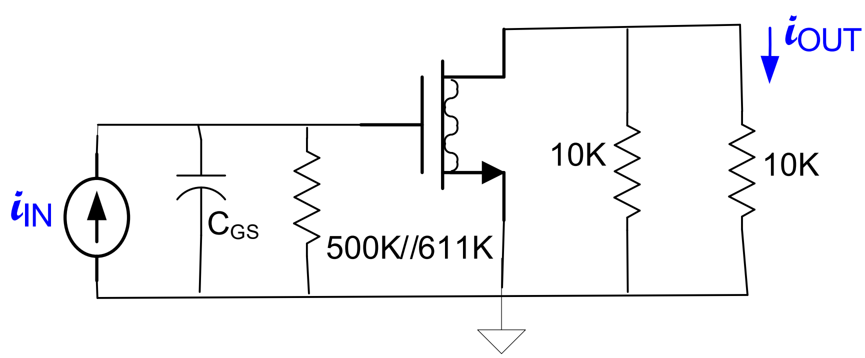
$$\therefore \quad A_v(s) = \frac{v_o}{v_i} = \frac{-g_m}{sC_L + g_o + G_L} \simeq \frac{-g_m}{sC_L + G_L}$$

Problem 2

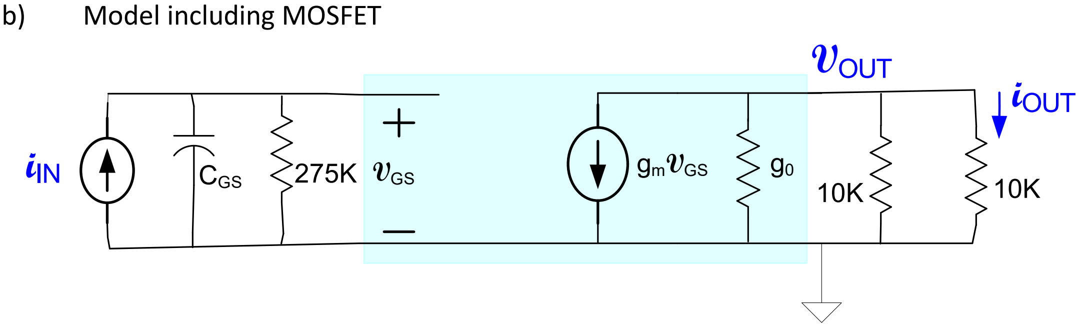
As originally posted, the transistor was not biased to operate in the saturation region. A change in the biasing has been made as shown below.



a) Small signal equivalent circuit including C_{GS} capacitor.



b) Model including MOSFET



Defining R_B=275K, $G_B=1/R_B$, and $G_L=1/R_L$=1/10K and summing currents at input and output nodes obtain equations

$$V_{GS}\left(sC_{GS}+G_B\right)=I_{IN}$$
$$V_{OUT}\left(g_o+G_L+G_L\right)+g_mV_{GS}=0$$
$$V_{OUT}G_L=I_{OUT}$$

Eliminating V_{OUT} and V_{GS} from these equations we obtain

$$\frac{I_{OUT}}{I_{IN}}=-\frac{g_m}{R_L\left(g_o+G_L+G_L\right)}\frac{1}{sC_{GS}+G_B}\simeq-\frac{R_Bg_m/2}{sR_BC_{GS}+1}$$

c) Want to obtain

$$\left|\frac{R_Bg_m/2}{j\omega R_BC_{GS}+1}\right|=1$$

Which can be written as

$$\frac{\left(R_Bg_m/2\right)^2}{1+\left(\omega R_BC_{GS}\right)^2}=1$$

Solving for angular frequency ω we obtain

$$\omega=\frac{\sqrt{\left(R_Bg_m/2\right)^2-1}}{R_BC_{GS}}\simeq\frac{g_m}{2C_{GS}}$$

It remains to obtain gm and CGS. Observe by voltage divider VGS=2.75V. So

$$g_m=\mu C_{OX}\frac{W}{L}\left(V_{GSQ}-V_{TH}\right)=2E-4$$

$$C_{GS}=C_{OX}WL=400\text{fF}$$

So unity gain frequency is 250M rad/sec or 40 MHz.

Problem 3 $I_{D2} = \left(\frac{W_2}{W_1}\frac{L_1}{L_2}\right) I_{D1} = (3)(50\mu A) = 150\mu A$

$V_o = 8V - I_{D2}\cdot R = 8V - (150\mu A)(25k) = 4.25V$

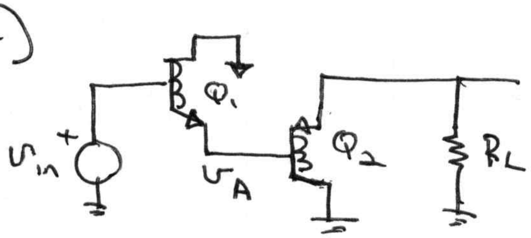
Problem 4 From Lecture Notes $A_v = -\frac{g_{m1}}{2 g_{o1}}\cdot\beta = -\frac{V_{AF}}{2V_t}\beta$

So $A_v \simeq \frac{-200}{50mV}\cdot 100 = -400{,}000$

Problem 5 Define $G_L = \frac{1}{R_L} = \frac{1}{1k\Omega}$

Observe this is a cascade of two CC stages

a)



$A_v = \frac{v_o}{v_A}\cdot\frac{v_A}{v_i}$

$\frac{v_o}{v_A} = \frac{+g_{m2}}{g_{m2}+G_L}$

$R_{in2} \simeq \beta_2 R_L$

$\frac{1}{R_{in2}} = \frac{G_L}{\beta_2}$

$\frac{v_A}{v_{in}} = \frac{+g_{m1}}{g_{m1}+\frac{G_L}{\beta_2}}$

$\therefore A_v = \left(\frac{g_{m2}}{g_{m2}+G_L}\right)\left(\frac{g_{m1}}{g_{m1}+\frac{G_L}{\beta_2}}\right) \simeq 1$

b) $v_{inQ} = 0V$, $V_{OQ} = V_{inQ} - 0.6 + 0.6 = 0V$

c) $R_{in} = r_{\pi 1} + \beta_1 R_{in2} = r_\pi + \beta_1\beta_2 R_L \simeq \beta_1\beta_2 R_L$

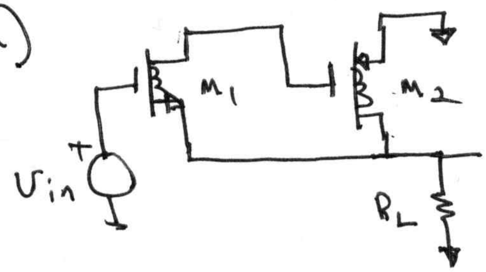
d) If current sources ideal

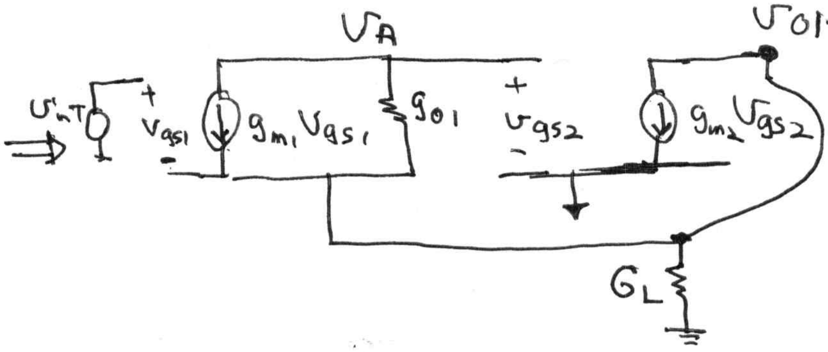
$v_{OMAX} = V_{CC}$

$v_{Omin} = V_{EE} + 0.6V$

Problem 6 Define $G_L = \frac{1}{R_L} = \frac{1}{1K\Omega}$

a)





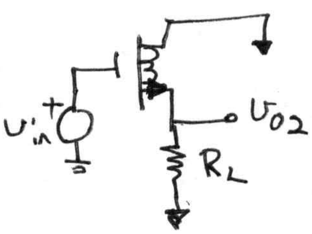
From KCL

$$V_{o1}(g_{o1}+G_L) + g_{m2}V_A = g_{m1}(V_{in}-V_{o1}) + g_{o1}V_A$$

$$V_A = V_{o1} - \frac{g_{m1}}{g_{o1}}(V_{in}-V_{o1})$$

eliminating V_A, obtain

$$\frac{V_{o1}}{V_{in}} = \frac{g_{m1}g_{o1} + g_{m1}g_{m2}}{g_{o1}[g_{m2}+G_L] + g_{m1}g_{m2}} \simeq \frac{g_{m1}g_{m2}}{g_{m1}g_{m2}}$$



Recognize as common drain amplifier

$$\therefore \frac{V_{o2}}{V_{in}} = \frac{g_{m1}}{g_{m1}+G_L}$$

b) $\frac{V_{o1}}{V_{in}} \simeq 1$ to find $\frac{V_{o2}}{V_{in}}$, need g_{m1}. First obtain I_{DQ}

$$I_{DQ} = \frac{\mu C_{ox} W}{2L}(V_{inQ} - I_{DQ}R - V_{TH})^2$$

solving this equation for I_D, obtain

$$I_{DQ} = 10\mu A \Rightarrow g_m = \sqrt{\mu C_{ox}\frac{W}{L} \cdot 2I_{DQ}} = 1E\text{-}4$$

thus

$$\frac{V_{o2}}{V_{in}} = \frac{E\text{-}4}{E\text{-}4 + \frac{1}{5K}} = \frac{1}{3}$$

c) Need to reduce $I_{B1} = 5\mu A$, $I_{B2} = 10\mu A$

For circuit on left

$$I_{B2} = \frac{\mu C_{ox} W}{2L}(V_{INQ} - V_{OQ} - V_{TH})^2$$

$$10\mu A = (E-4)\left(\frac{10}{4}\right)(1V - V_{OQ} - .75V)^2$$

solving, obtain $V_{OQ} = 50mV$

For circuit on right, found in part b), $I_{DQ} = 10\mu A$

$\therefore V_{OQ} = (10\mu A)(5k) = 50mV$

d) For $V_{INQ} = 4V$, circuit on left has

$V_{OQ} = 3.05V$

For circuit on right, must again solve

$$I_{DQ} = \frac{\mu C_{ox} W}{2L}(V_{inQ} - I_{DQ}R - V_{TH})^2 \quad \text{for } I_{DQ}$$

with $V_{inQ} = 4V$; obtain $I_{DQ} = 0.4mA$

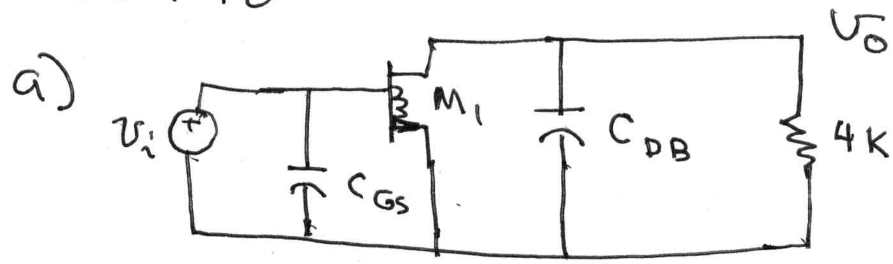
so $V_{OQ} = (I_{DQ})(5k) = 2V$

Problem 7 From Lecture Slides

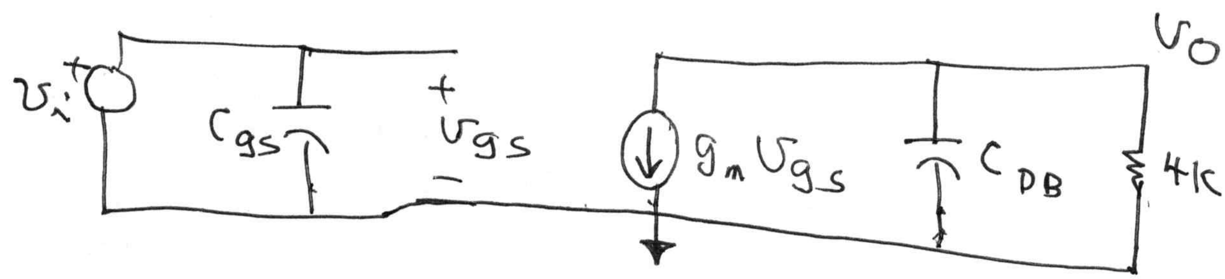
$$A_v \simeq -\frac{g_{m1}}{g_{o1}} = -\frac{\frac{I_{CQ}}{V_t}}{\frac{I_{CQ}}{V_{AF}}} = \frac{V_{AF}}{V_t} = -\frac{100}{25mV} = -4000$$

Problem 8

a)



b)



summing currents at output node with $G_L = \frac{1}{4K}$

$$v_o \left(s C_{DB} + G_L \right) + g_m v_i = 0$$

$$\therefore \quad \frac{v_o}{v_{in}} = \frac{-g_m}{s C_{DB} + G_L}$$

c) $\omega_{3dB} = \frac{G_L}{C_{DB}} \Rightarrow f_{3dB} = \frac{1}{(2\pi) R_L C_{DB}}$

$$C_{DB} = C_{SWD} \cdot 52\mu + C_{BOT} \cdot 120\mu^2$$

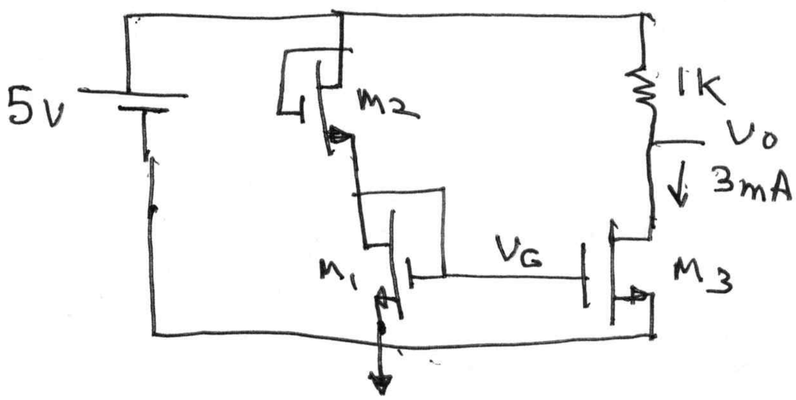
From table $C_{SWD} = 212\ af/\mu$

$C_{BOT} = 942\ af/\mu^2$

$$\therefore C_{DB} = 11.0\ fF + 113\ fF = 124\ fF$$

So $f_{3dB} = \frac{1}{(2\pi)(4K)(124fF)} = 320\ MHz$

Problem 9 One solution



Since $V_o = 2V$, want $V_G < 2V + V_{TH}$ to maintain saturation of M3.

So will set $V_G = 2V$

$$I_{D3} = \frac{\mu C_{ox} W}{2L} (2 - .75)^2$$

$$3E-3 = \frac{(E-4)}{2} \cdot \frac{W_3}{L_L} (1.25)^2$$

$\therefore \frac{W_3}{L_3} = 38$. Let $L_3 = 1\mu$, $W_3 = 38\mu$

Let $\frac{W_1}{L_1} = 38$ so unity mirror gain
Let $L_1 = 1\mu$, $W_1 = 38\mu$

$\therefore I_{D1} = 3mA$

Consider now M_2, which also has $I_D = 3mA$

$$\therefore \quad 3mA = \frac{\mu C_{ox} W_2}{2 L_2} (5 - V_G - V_{TH})^2$$

$$3mA = \frac{(E-4)}{2} \frac{W_2}{L_2} (5 - 2 - .75)^2$$

solving

$\frac{W_2}{L_2} = 11.8$ Let $L_2 = 10\mu$, $W_2 = 118\mu$